ph260 Theoretical Physics 2 — workshop 6

1. Fourier transforms.
Determine the Fourier transforms of the following functions:

a. $f(x) = \{-1 \quad (-\pi < x < 0); \qquad +1 \quad (0 < x < \pi); \qquad 0 \quad (\text{otherwise})\}$.
b. $f(t) = \mathrm{e}^{-kt} g(t)$, where $g(t) = \{1 \quad (0 \leq t \leq b); \qquad 0 \quad (\text{otherwise})\}$.

2. Calculating a spectrum.
A complex time-domain signal is measured in an experiment. After an initial fast decay with time constant k_1, it decays exponentially with time constant k_2 and has two cosine waves superimposed whose periods are $1/c_1$ and $1/c_2$. For experimental reasons, only the time range from t_1 to t_2 can be observed. Find the function $f(t)$ that describes this behaviour and the corresponding spectrum $F(\omega)$ (You will get four terms for each combination of $k_{1,2}$ and $c_{1,2}$ – they all have the same form; so exploit the symmetry to keep the equations simple.). If you can get hold of a computer with mathematical software on it, plot the real and imaginary parts of both $f(t)$ and $F(\omega)$ and see how changes to the parameters k_1, k_2, c_1, c_2, t_1, t_2 affect the spectrum. In particular, investigate the following:

a. What is the effect of cutting off the decay before it is close to zero (*i.e.* too small t_2)?
b. What is the effect of starting the time domain at $t_1 > 0$?
c. What is the minimum ratio $\frac{k_1}{k_2}$ for which the spectral contributions from the two decays remain discernible?
d. What is the relative minimum frequency difference $\frac{c_1 - c_2}{c_1}$ of the two waves to ensure they come out as distinct peak maxima in the spectrum?

Acknowledgement.
Example 1a is stolen from *ML Boas; Mathematical Methods in the Physical Sciences, John Wiley, New York (USA)* ^{2}*1983.*

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