ph260 Theoretical Physics 2 — workshop 5

1. Wave equation.
In the lecture, we have worked out the equation of motion of a *plucked* string, *i.e.* a vibrating string of length l whose motion begins with the string initially at rest but in a non-equilibrium position. Its boundary conditions were $y(x,0) = \begin{cases} \frac{2hx}{l} \; (0 \le x \le \frac{l}{2}) \\ \frac{2h(l-x)}{l} \; (\frac{l}{2} \le x \le l) \end{cases}$ and $\left.\frac{\partial y(x,t)}{\partial t}\right|_{t=0} = 0$.

If the string is *hit* rather than plucked, *e.g.* in a piano, the boundary conditions are opposite: The string is in relaxed position at the beginning, but due to the impact of the hammer, it is at maximum velocity. This initial velocity is proportional to the distance from the ends of the string. Cast the boundary conditions in mathematical formalism, choose the correct general solution of the wave equation, apply the boundary conditions, and find the series solution of $y(x,t)$ of the piano string.

2. Cartesian drum.
Drums are usually circular – for mathematically sound reasons as we'll see. Write down the wave equation $z(x,y,t) = \ldots$ for a "Cartesian drum", *i.e.* a drum with a membrane fixed to a rectangular frame of size $a \times b$. Separate the equation into a spatial and a temporal part and find the general solutions. Work out the boundary conditions of the problem (assuming that the drum is hit with a drumstick to start the oscillation), decide which of the general solutions can be discarded because they conflict with the BCs, and write down a linear combination of the remaining ones as a preliminary solution of this wave equation. Will the Cartesian drum's sound be harmonic?

Acknowledgement.
Example 1 is stolen from *ML Boas; Mathematical Methods in the Physical Sciences, John Wiley, New York (USA)* ^{2}*1983.*

rw/031027